by strain and the initial kinetic energy of the liner is independent of R_0 but is governed by the ratio R_0/h_0, the value of the initial compression rate $\dot{R}(0)$, and the magnitude of the dynamic yield point. Estimates executed by means of (11) and (12) are in good agreement with the computation results according to the equation of motion.

In conclusion, we note the following.

1. Comparison of the results of calculations with the experiments conducted confirms the feasibility of using relationships (2), (3), and (4) for the determination of the characteristics of motion of a liner and the computation of energy loss by deformation.

2. The magnitude of the dynamic limit point of aluminum, in experiments with liners under conditions typical for electromagnetic acceleration, was a constant and uniform $2 \cdot 10^8$ Pa.

3. The energy loss by deformation, determined according to expression (12), in the interval $R_0/h_0 = 20$–60 for $R(0) = (0.25$–$1.5) \cdot 10^3$ m/sec, coincides well with the estimate from the solution of equation (10).

LITERATURE CITED

1. V. P. Knyazev and G. A. Shneerson, "Investigation of fast broadening of thin-walled metal cylinders in a strong magnetic field," Zh. Tekh. Fiz., 40, No. 2 (1970).
2. F. Herlach and J. E. Kennedy, "The dynamics of imploding liners in magnetic flux compression experiments," Appl. Phys., 6 (1973).
3. V. R. Regel', A. I. Slutsker, and É. E. Tomashevskii, "Kinetic nature of the strength of solids," Usp. Fiz. Nauk, 106, No. 2 (1972).
4. S. N. Zhurkov and É. E. Tomashevskii, "Time dependence of the strength for different loading modes," Certain Problems of the Strength of a Solid [in Russian], Izd. Akad. Nauk SSSR, Moscow (1959).
5. M. S. Kachan and Yu. A. Trishin, "Tensile stresses in a target during the collision of solids," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1977).
6. M. J. Forrestal and H. C. Walling, "Axisymmetric plastic response of rings to short-duration pressure pulses," AIAA J., 10, No. 10 (1972).
7. Ya. B. Zel'dovich and A. D. Myshkis, Elements of Applied Mathematics [in Russian], Nauka, Moscow (1967).
8. D. Bland, Theory of Linear Viscoelasticity [Russian translation], Mir, Moscow (1965).
9. W. Prager, Introduction to the Mechanics of Continuous Media [Russian translation], IL, Moscow (1963).

RESIDUAL STRESSES AND VISCOSITY IN THE HIGH-SPEED DEFORMATION OF METALS

N. N. Sergeev-Al'bov

UDC 539.376

By formulating a numerical experiment, the residual stresses occurring in the surface layer of a metal after the passage of a pressure pulse are studied in this paper. Their magnitude as a function of the magnitude and rate of traversal of the acting pressure pulse is studied. The stressed behavior of the metal is described by Maxwell equations of a linear viscoelastic medium [1]. Taking account of plastic phenomena occurs in this model because of the introduction of a nonlinear dependence of the relaxation time on the tangential stress intensity.

To describe metal behavior under high-speed strain, a viscous incompressible fluid model is often used. Within the framework of this model, the viscosity of metals during collisions in the explosive welding mode is investigated in [2]. Here the model equation utilized in [2] and the Maxwell model are compared in an example of numerical results.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 116-121, March-April, 1983. Original article submitted February 9, 1982.

0021-8944/83/2402-0242$07.50

1. Let the stressed behavior of a metal layer moving at a constant subsonic velocity w along the x axis of a plane of the variables (x, y) be described by the Maxwell model of a linear viscoelastic medium with a relaxation time nonlinearly dependent on the stress state of the metal. The system of equations will have the form [1]

$$\rho_0 \frac{du}{dt} - \frac{\partial\sigma_{11}}{\partial x} - \frac{\partial\sigma_{12}}{\partial y} = 0, \quad \rho_0 \frac{dv}{dt} - \frac{\partial\sigma_{12}}{\partial x} - \frac{\partial\sigma_{22}}{\partial y} = 0, \tag{1.1}$$
$$\frac{d(\sigma_{11}-\sigma)}{dt} - \frac{4}{3}\rho_0 b_0^2 \frac{\partial u}{\partial x} + \frac{2}{3}\rho_0 b_0^2 \frac{\partial v}{\partial y} = -\frac{\sigma_{11}-\sigma}{\tau},$$
$$\frac{d(\sigma_{22}-\sigma)}{dt} + \frac{2}{3}\rho_0 b_0^2 \frac{\partial u}{\partial x} - \frac{4}{3}\rho_0 b_0^2 \frac{\partial v}{\partial y} = -\frac{\sigma_{22}-\sigma}{\tau},$$
$$\frac{d\sigma}{dt} - \rho_0\left(c_0^2 - \frac{4}{3}b_0^2\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \quad \frac{d\sigma_{12}}{dt} - \rho_0 b_0^2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = -\frac{\sigma_{12}}{\tau},$$

where

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + w\frac{\partial}{\partial x}; \quad \sigma = \frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3}; \quad w < b_0 < c_0;$$

$(u + w, v)$ are the displacement velocity vector components of points of the medium; σ_{ij}, stress tensor components; ρ_0, density of the medium; and c_0, b_0, longitudinal and transverse speeds of sound.

The interpolation formula [3]

$$\tau = \tau_0\left(\frac{\rho_0 b_0^2}{In_0}\right)^{n-1}\exp\left(\mu\frac{\Phi}{RT_0}\right), \quad n^{-1} = n_1\left[\left(\frac{T_0}{\theta n_2} - 1\right)^2 + n_3\right],$$
$$\Phi(T_0) = c_0^2\frac{T_0}{\theta n_4}\left(1 - \frac{T_0}{\theta n_5}\right)n, \quad \theta = 315\,\mathrm{K}, \quad T_0 = 300\,\mathrm{K}, \quad \mu = 6354\,\mathrm{g/mole}, \tag{1.2}$$
$$R = 8.31\ \mathrm{J/deg\cdot mole}$$

is taken for the relaxation time. Here $I = \sqrt{((\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6\sigma_{12}^2)/2}$ is the tangential stress intensity. The interpolation constants of this formula are presented in Table 1. The metal layer on the (x, y) plane is given in the form

$$S = \{x, y \mid -\infty < x < +\infty, \ 0 \leqslant y \leqslant l_2\}.$$

The pressure pulse is modeled as follows. On the layer boundary $y = l_2$ for $x_1 \leqslant x \leqslant x_2$. The rest of the upper and the lower boundaries of the layer are stress-free. At the initial time (t = 0), the velocity and stress field in the metal layer are zero.

The computational domain in the numerical experiment is the finite rectangle $\Pi = \{x, y \mid 0 \leqslant x \leqslant l_1, \ 0 \leqslant y \leqslant l_2\}$ (l_1 = 65 mm, l_2 = 8 mm.

This rectangle Π is divided into rectangular difference cells with the sides h_1 and h_2. A number (n, m) corresponds to each cell, where n = 0, 1, ..., N1, N1 + 1, ..., N2, ..., N3; m = 0, 1, ..., M1; N1 = 11; N2 = 20; N3 = 49 and M1 = 13. Then $h_1 = l_1/(1 + N3)$, $= l_2/(1 + M1)$.

At the initial time t = 0 there holds in the rectangle Π

$$u = v = 0, \ \sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{12} = 0, \tag{1.3}$$

and the following conditions are satisfied on its boundaries

$$\text{for } y = 0 \ \sigma_{12} = \sigma_{22} = 0,$$
$$\text{for } y = l_2 \ \sigma_{12} = 0, \ \sigma_{22} = -p \ \text{for } x_1 \leqslant x \leqslant x_2 \ (x_1 = N1 \times h_1, x_2 =$$
$$= N2 \times h_2), \ \sigma_{22} = 0 \text{ for all other } x, \tag{1.4}$$
$$\text{for } x = 0 \ [\sigma_{11} - \rho_0 c_0 u] = 0, \ v = 0,$$
$$\text{for } x = l_1 \ [\sigma_{11} + \rho_0 c_0 u] = 0, \ \sigma_{12} = 0.$$

To find the coordinates of points of the metal layer during its deformation we use the equations

$$\partial\xi/\partial t + w\partial\xi/\partial x = u + w, \qquad \partial\eta/\partial t + w\partial\eta/\partial x = v. \tag{1.5}$$

A numerical algorithm for the computation of the problem (1.1)-(1.5) is constructed on the basis of the S. K. Godunov difference method [4], which is described in detail in [5] in application to this problem.

2. In the stationary case, the following relationships are satisfied for the system (1.1) on the double real characteristic of the streamline y = const [6]

$$\frac{\partial}{\partial x}\left[\sigma_{11}-\frac{4\rho_0 b_0^2}{w}\left(\frac{c_0^2-b_0^2}{c_0^2}\right)u-\left(\frac{c_0^2-2b_0^2}{c_0^2}\right)\sigma_{22}\right]=-\frac{\left(\sigma_{11}-\sigma-\left(\frac{c_0^2-2b_0^2}{c_0^2}\right)(\sigma_{22}-\sigma)\right)}{\tau w}, \tag{2.1}$$

$$\frac{\partial}{\partial x}\left[\sigma_{33}-\frac{c_0^2-2b_0^2}{c_0^2}\left(\sigma_{22}+\frac{2\rho_0 b_0^2}{w}u\right)\right]=-\left[\sigma_{33}-\sigma-\left(\frac{c_0^2-2b_0^2}{c_0^2}\right)(\sigma_{33}-\sigma)\right]\Big/(\tau w).$$

The function τ is an abruptly changing function of the state of the substance; thus $\tau = \infty$ in the elastic zone, while $\tau \simeq 1$ μsec in the zone of developed plastic deformations. Therefore, additives determined by the right sides of the relationships (2.1) remain in the components σ_{11} and σ_{33} after the metal has passed through this zone.

The presence of such properties of the system of equations being studied permits the computation of the residual stresses that originate in a metal layer because of pressure pulse motion over this layer.

Stress and velocity fields in copper (Table 2) are studied as a function of the velocity w of pressure pulse motion and of its magnitude p. We shall divide the desired solution of the viscoelastic equations over two components, elastic and viscous:

$$u = u^{\mathrm{ela}} + u^{\mathrm{v}},\ v = v^{\mathrm{ela}} + v^{\mathrm{v}},$$
$$\sigma_{ij} = \sigma_{ij}^{\mathrm{ela}} + \sigma_{ij}^{\mathrm{v}},\ \xi = \xi^{\mathrm{ela}} + \xi^{\mathrm{v}},\ \eta = \eta^{\mathrm{ela}} + \eta^{\mathrm{v}}.$$

The elastic part of the solution satisfies the problem (1.1)-(1.4) under the condition that the relaxation time τ is infinity in the system (1.1).

The computation is performed in steps in time up to the build-up of the stress and velocity fields. The times $t \geqslant 2000h_3$ correspond to the build-up of the physical quantities.

Graphs of the stress σ_{11} are presented in Fig. 1 for fixed values of x(N = 24, 30, 36, curves 1-3, respectively) as a function of y(M = 0, 1, ..., 13) for a problem with the parameters $p = 5\cdot 10^8$ Pa, w = 1.8 km/sec.

For σ_{11} and σ_{33} different from zero, the stress tensor components σ_{12} and σ_{22} are almost zero, where the σ_{11} have negative values on the side of the layer boundary on which the pressure pulse acted.

Therefore, a layer of residual compressive stresses forms in a near-boundary strip of the material (stressed layer). Its magnitude depends on the material yield point, which is taken into account in this case by giving the relaxation time dependence on the tangential stress intensity, as well as on the magnitude of the parameters p and w.

Diagrams of the stress σ_{11} are presented in Fig. 2 for fixed x (N = 28) as a function of y for $p = 5\cdot 10^8$, 10^9 Pa (curves 1 and 2, respectively) and w = 1.8 km/sec.

3. To describe the behavior of metals under high-speed deformation, the model of a

TABLE 1

τ_0	n_0	n_1	n_2	n_3	n_4	n_5
2,4 μsec	0,000196	0,0184	0,955	1,902	0,00014	7,22

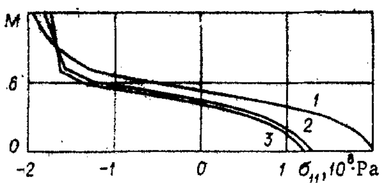

Fig. 1

TABLE 2

ρ_0	c_0	b_0
8,9 g/cm³	4,65 km/sec	2,14 km/sec

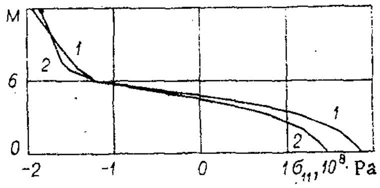

Fig. 2

viscous incompressible fluid is often utilized. Within the framework of this model, the viscosity of metals during their collision in the explosive welding mode is investigated in [2].

Let us compare the model equation used in [2], and the Maxwell viscoelastic equations in an example of the numerically computed problem.

Interpolation formulas for the relaxation time τ are presented in [7]. These interpolation formulas were constructed by determining the coefficient of viscosity in metals during explosive welding [2]. Here the dependence between the coefficient of viscosity μ and the relaxation time τ was taken into account [7]:

$$\mu = \rho_0 b_0^2 \tau. \tag{3.1}$$

We have a value of the relaxation time τ at each computational point, and hence, according to (3.1), the value of the coefficient of viscosity μ as well. A numerical computation permits determination of the viscous horizontal displacements of the particles of the metal layer. They are determined by the equality $\xi^{v} = \xi - \xi^{ela}$.

Under stationarity conditions for the deformation process, the first and subsequent equations of the system (1.1) for the viscous components have the form

$$\rho_0 w \frac{\partial u^{v}}{\partial x} = \frac{\partial \sigma_{11}^{v}}{\partial x} + \frac{\partial \sigma_{12}^{v}}{\partial y}; \tag{3.2}$$

$$w \frac{\partial \sigma_{12}^{v}}{\partial x} - \rho_0 b_0^2 \left(\frac{\partial u^{v}}{\partial y} + \frac{\partial v^{v}}{\partial x} \right) = - \frac{\sigma_{12}^{v}}{\tau} - \frac{\sigma_{12}^{ela}}{\tau}. \tag{3.3}$$

Equation (3.3) can be rewritten in the form

$$\sigma_{12}^{v} = \mu \left(\frac{\partial u^{v}}{\partial y} + \frac{\partial v^{v}}{\partial x} \right) - \tau w \frac{\partial \sigma_{12}^{v}}{\partial x} - \sigma_{12}^{ela}.$$

Substituting the equation obtained in (3.2) (under the assumption that τ = const), we obtain

$$\rho_0 w \frac{\partial u^{v}}{\partial x} - \frac{\partial \sigma_{11}^{v}}{\partial x} = \mu \frac{\partial^2 u^{v}}{\partial y^2} + Q, \text{where} Q = \mu \frac{\partial^2 v^{v}}{\partial x \partial y} - \tau w \frac{\partial^2 \sigma_{12}^{v}}{\partial x \partial y} - \frac{\partial \sigma_{12}^{ela}}{\partial y}. \tag{3.4}$$

We integrate this equation with respect to x between $-\infty$ and $+\infty$:

$$\rho_0 w \left[u_{\infty}^{v} - u_{-\infty}^{v} \right] - \left[\sigma_{11\infty}^{v} - \sigma_{11-\infty}^{v} \right] = \mu w \frac{d^2 z}{dy^2} + \int_{-\infty}^{+\infty} Q dx,$$

where $z(y) = \frac{1}{w} \int_{-\infty}^{+\infty} u^{v}(x, y) dx$ is the horizontal viscous displacement

$$u_{-\infty}^{v} = 0; \quad \sigma_{11-\infty}^{v} = 0; \quad \left. \frac{\partial \sigma_{12}^{v}}{\partial y} \right|_{-\infty} = 0; \quad \left. \frac{\partial v^{v}}{\partial x} \right|_{-\infty} = 0.$$

We have from the results of the numerical computation

$$\left. \frac{\partial \sigma_{12}^{v}}{\partial y} \right|_{+\infty} \simeq 0, \left. \frac{\partial v^{v}}{\partial y} \right|_{+\infty} \simeq 0, \int_{-\infty}^{+\infty} \frac{\partial \sigma_{12}^{ela}}{\partial y} dx = \rho_0 w u_{\infty}^{ela} - \sigma_{11\infty}^{ela} \simeq 0.$$

This means that the magnitude of the integral of Q can be considered approximately zero.

Then the horizontal viscous displacement of particles of the metal layer z(y) is determined from the ordinary differential equation

$$\rho_0 w \overset{\vee}{u}_\infty - \overset{\vee}{\sigma}_{11\infty} = \mu w \frac{d^2 z}{\partial y^2} \tag{3.5}$$

which can be obtained from the model equation

$$\rho_0 w \frac{\partial \overset{\vee}{u}}{\partial x} - \frac{\partial \overset{\vee}{\sigma}_{11}}{\partial x} = \mu \frac{\partial^2 \overset{\vee}{u}}{\partial y^2} \tag{3.6}$$

by integrating with respect to x between $-\infty$ and $+\infty$. In turn (3.6) can be obtained from (3.4) if the quantity Q is neglected therein.

Equation (3.6) differs from the equation for the steady motion of a viscous incompressible fluid

$$\rho_0 w \partial u/\partial x = \mu \partial^2 u/\partial y^2, \tag{3.7}$$

used in [7] by the term $\partial \overset{\vee}{\sigma}_{11}/\partial x$.

It will be shown below that taking account of this term substantially affects the horizontal viscous displacement of the metal layer particles.

It follows from (1.4) and (2.3) that $\overset{\vee}{\sigma}_{11\infty} + \rho_0 c_0 \overset{\vee}{u}_\infty = 0$, then (3.5) is converted to the form

$$\frac{1}{b_0^2 \tau}\left(1 + \frac{c_0}{w}\right) \overset{\vee}{u}_\infty = \frac{d^2 z}{\partial y^2},$$

and (3.7) to the form

$$\frac{1}{b_0^2 \tau} \overset{\vee}{u}_\infty = \frac{d^2 z}{dy^2}.$$

Let us use the notation

$$A = \begin{cases} \dfrac{1}{b_0^2 \tau}\left(1 + \dfrac{c_0}{w}\right) u_\infty & \text{for Eq. (3.6)} \\ \dfrac{1}{b_0^2 \tau} \overset{\vee}{u}_\infty & \text{for Eq. (3.7)}. \end{cases}$$

Then (3.6) and (3.7) can be rewritten in the form

$$d^2 z/dy^2 = A. \tag{3.8}$$

Following [2], we consider the motion of the metal layer as an ideal fluid jet with constant pressure. The velocity along the free surface is w here. After the pressure pulse has acted on it, the jet rotates through an angle γ. Then the law of conservation of momentum is not satisfied along the x axis; the momentum is greater prior to jet rotation than after rotation through the angle γ. As in [2], we shall assume the presence of a pulse source which will shape a viscous fluid jet moving at a lower speed than the velocity w. From the law of conservation of momentum we find the impulse $J = \rho_0 w l_2 (1 - \cos \gamma)$ along the x axis.

Then the dissipated velocity $\overset{\vee}{u}_\infty$ as $x \to \infty$ will be determined in the form $\overset{\vee}{u}_\infty = J/\rho_0 l_2 = w(1 - \cos \gamma) \approx w\gamma^2/2$.

From the law of conservation of vertical impulse it follows that $\sigma_{22}^{\Gamma} l = \rho_0 l_2 w^2 \sin \gamma$, where $\sigma_{22}^{\Gamma} = -p$; l is the length of the part of the layer on which the pressure pulse acts. We hence obtain the relationship $\sin \gamma = -pl/(\rho_0 l_2 w^2)$.

Let us put

$$z\big|_{y=0} = 0, \quad \frac{dz}{dy}\bigg|_{y=l_2} = 0. \tag{3.9}$$

Let us solve (3.8) under the condition (3.9). We obtain that $z(y) = Ay^2/2 - Al_2 y$ is the desired horizontal displacement.

We evaluate the magnitude of the integrals of the viscous displacement (over the height of the metal layer) obtained from the Maxwell model equation (3.6):

$$S_M = \int_0^{l_2} z(y)\, dy = -\frac{l_2^3}{3} \frac{\overset{\vee}{u}_\infty}{b_0^2 \tau}\left(1 + \frac{c_0}{w}\right);$$

TABLE 3

w, km/sec	1,8	1,8	1,2
p, 10^{-8}Pa	7,5	5	5
S_{comp}, mm	—0,54	—0,35	—1,04
S_M, mm	—0,41	—0,15	—0,48
S_V, mm	—0,11	—0,04	—0,1
$\bar{\mu}_{comp}$, P	$1{,}6\cdot10^5$	$2{,}4\cdot10^5$	$3{,}7\cdot10^5$
μ_M, P	$1{,}5\cdot10^5$	10^5	$1{,}6\cdot10^5$
μ_V, P	$4{,}1\cdot10^4$	$2{,}9\cdot10^4$	$3{,}3\cdot10^4$

from the equation for the steady motion of a viscous incompressible fluid (3.7):

$$S_B = \int_0^{l_2} z(y)\,dy = -\frac{l_2^3}{3}\frac{u_\infty^V}{b_0^2\tau},$$

and also the total displacement obtained from the numerical computation

$$S_{comp} = \sum_{m=0}^{M1} \tilde{z}_{m+1/2} h_2,$$

where $\tilde{z}_{m+1/2} = \tilde{z}(h_2(m + 1/2)) = \xi_{n,m} - \xi_{n,m}^{ela}$.

The values of the quantities S_M, S_V, S_{comp} are presented in Table 3. The magnitude of the relaxation time τ, meaning according to (3.1), the magnitude of the coefficient of viscosity μ also, can be calculated from the values of S_M and S_V. Values of the three coefficients of viscosity μ_M, μ_V and μ_{comp} are presented in Table 3. The relaxation time τ is rarely a variable function in a numerical computation, hence μ_{comp} evaluated according to the mean value of τ in the plastic zone is presented in Table 3.

Analysis of Table 3 shows that the coefficient of viscosity μ_V computed from the viscous incompressible fluid model differs from the coefficient of viscosity μ_{comp} obtained by formulation of a numerical experiment within the framework of the Maxwell viscoelastic model. On the other hand, the coefficient of viscosity μ_M computed from the Maxwell model equation (3.6) agrees in orders of magnitude with the computed coefficient of viscosity μ_{comp}. The model equation (3.6) differs from (3.7) for steady viscous incompressible fluid motion by the term $\partial\sigma_1^Y/\partial x$. Compressive residual stresses σ_1^Y are formed in the metal as a result of its passage through the plastic deformation zone. The presence of residual stresses results in a larger horizontal displacement of the metal layer particles than the analogous displacement given by the viscous incompressible fluid model, which means different orders of magnitude of the values of the viscosity coefficient.

The author is grateful to S. K. Godunov for formulating and discussing the problem.

LITERATURE CITED

1. S. K. Godunov, Elements of the Mechanics of a Continuous Medium [in Russian], Nauka, Moscow (1978).
2. S. K. Godunov, A. A. Deribas, et al., "Investigation of the viscosity of metals under high-speed collisions," Fiz. Goreniya Vzryva, No. 1 (1971).
3. N. S. Kozin and N. K. Kuz'mina, "Shock adiabats and weak shock profiles in metals," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1977).
4. S. K. Godunov, A. V. Zabrodin, M. Ya. Ivanov, A. N. Kraiko, and G. P. Prokopov, Numerical Solution of Multidimensional Gasdynamic Problems [in Russian], Nauka, Moscow (1976).
5. N. N. Sergeev-Al'bov, Numerical Computation of Residual Stresses [in Russian], Preprint No. 212, Vychisl. Tsentr. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1980).

6. S. K. Godunov and N. N. Sergeev-Al'bov, "Equations of linear elasticity theory with point Maxwell sources of stress relaxation," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1977).
7. S. K. Godunov, A. F. Demchuk, et al., "Interpolation formulas for the dependence of the Maxwell viscosity," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1974).

SOME CONTACT PROBLEMS IN STEADY-STATE NONLINEAR CREEP IN CASES WITH THIN COVERINGS

V. M. Aleksandrov and E. V. Kovalenko UDC 539.376

1. We shall first give some fundamental relations in the nonlinear theory of creep for the case of plane deformation which are necessary for the rest of our discussion [1]:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0,\quad \frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0,\tag{1.1}$$
$$\frac{\partial^2\dot{\varepsilon}_x}{\partial y^2}+\frac{\partial^2\dot{\varepsilon}_y}{\partial x^2}=2\frac{\partial^2\dot{\gamma}_{xy}}{\partial x\,\partial y},$$
$$\dot{\varepsilon}_x=A\sigma_i^{m-1}[(1-\nu)\sigma_x-\nu\sigma_y],\ \dot{\varepsilon}_y=A\sigma_i^{m-1}[(1-\nu)\sigma_y-\nu\sigma_x],$$
$$\dot{\gamma}_{xy}=A\sigma_i^{m-1}\tau_{xy}\quad(m\geqslant 1),$$
$$\sigma_i=\frac{1}{\sqrt{6}}\sqrt{(\sigma_x-\sigma_y)^2+[(1-\nu)\sigma_x-\nu\sigma_y]^2+[(1-\nu)\sigma_y-\nu\sigma_x]^2+6\tau_{xy}^2},$$

where A is the creep modulus and ν is the Poisson coefficient.

Now we consider the solutions of some problems in the equilibrium of a thin layer† $(|x|<\infty,\ 0\leqslant y\leqslant h)$, whose physical and mechanical properties can be described by the system of equations (1.1). Suppose that the boundary conditions on the faces of the layer have the form

$$\tau_{xy}=0\ (y=0,\ y=h),\ \sigma_y=-p^*(x,\ t)\quad(y=h),\tag{1.2}$$
$$p^*(x,t)=p(x,t)\quad(|x|\leqslant a),\ p^*=0\quad(|x|>a),\ \dot{v}=B\dot{\sigma}_y\quad(y=0).$$

Here t is time; v is displacement along the y axis; B is some linear operator whose form will be indicated below, or

$$\tau_{xy}=0\ (y=h),\ \sigma_y=-p^*(x,\ t)\ (y=h),\tag{1.3}$$
$$\dot{u}=0\,(y=0),\ \dot{v}=B\dot{\sigma}_y\quad(y=0),$$

u is displacement along the x axis.

Taking account, furthermore, of the fact that the layer is thin, we see that instead of the condition of compatibility of the rates of deformation defined by the third formula in (1.1), we can take

$$\tau_{xy}=f_1(x)+yf_2(x).\tag{1.4}$$

Then the approximate solutions of the boundary-value problems (1.1)-(1.4) can be written in the form [2, 3]

$$\tau_{xy}=\sigma_x=0,\ \sigma_y=-p^*(x,\ t),$$
$$\dot{\varepsilon}_y=-A(1-\nu)[(1-\nu+\nu^2)/3]^{(m-1)/2}|p^*(x,t)|^m\operatorname{sgn}p^*(x,t);\tag{1.5}$$
$$\sigma_x=-\nu(1-\nu)^{-1}p^*(x,t),\ \sigma_y=-p^*(x,t),\ \tau_{xy}=-\nu(1-\nu)^{-1}\times$$
$$\times(h-y)[p^*(x,t)]',\ \dot{\varepsilon}_y=-A3^{(1-m)/2}\left[(1-2\nu)(1-\nu)^{-1}\right]^m|p^*(x,t)|^m\operatorname{sgn}p^*(x,t).\tag{1.6}$$

†A layer will be considered thin if the length $2a$ of its actively loaded segment is small in comparison with the thickness h.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 121-130, March-April, 1983. Original article submitted January 13, 1982.

 0021-8944/83/2402-0248$07.50